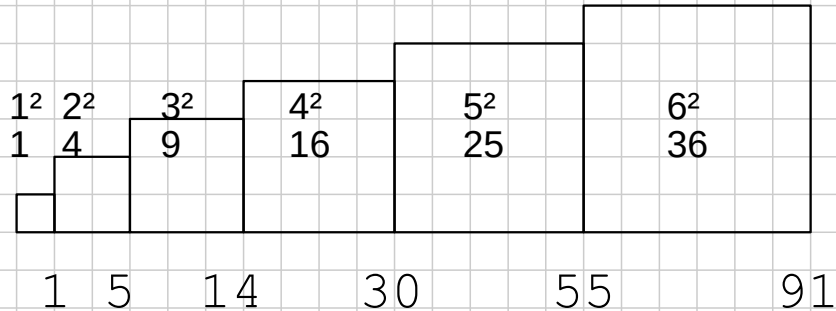


# Somma dei quadrati dei numeri interi vista come figura piana

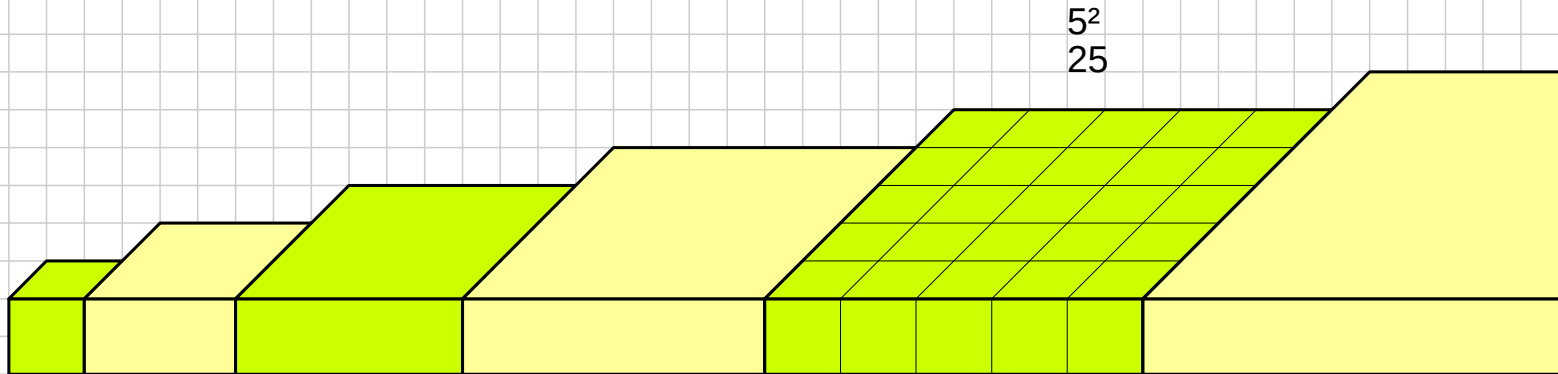


$$1^2 + 2^2 + 3^2 + \dots + n^2 = S_n$$

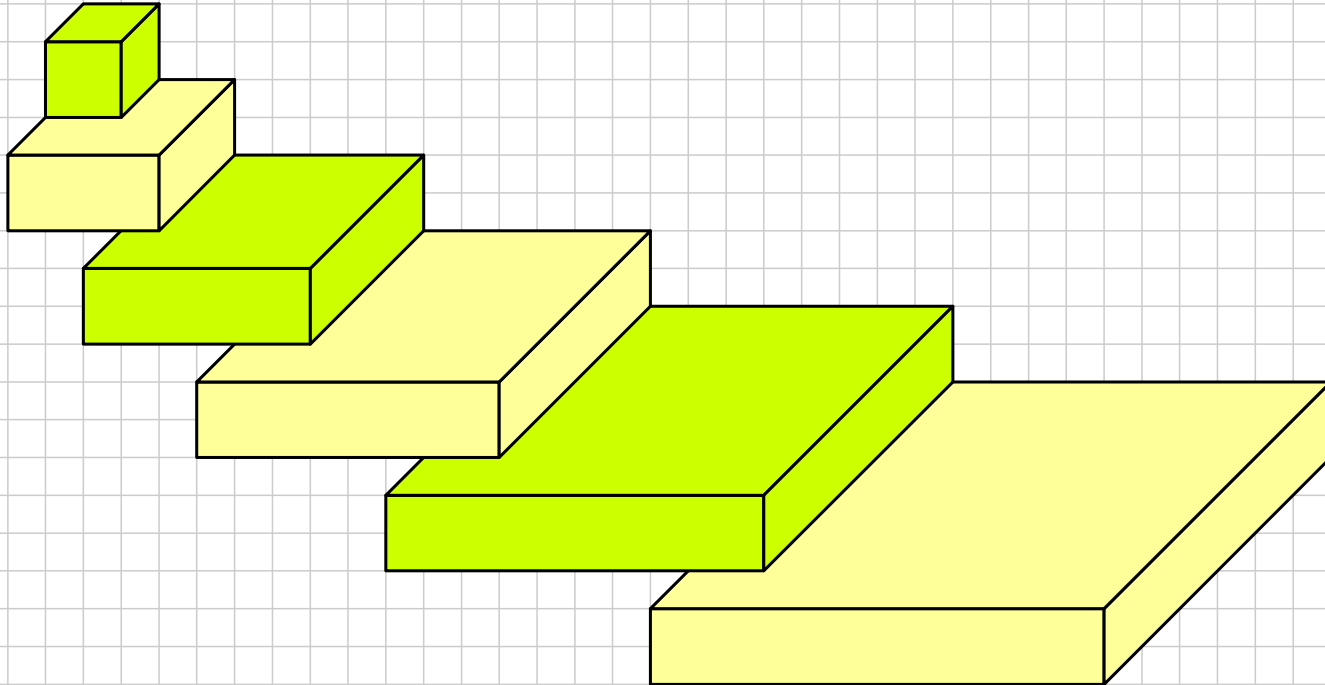
$$S(n+1) = S_n + (n+1)^2$$

$$S(n-1) = S_n - n^2$$

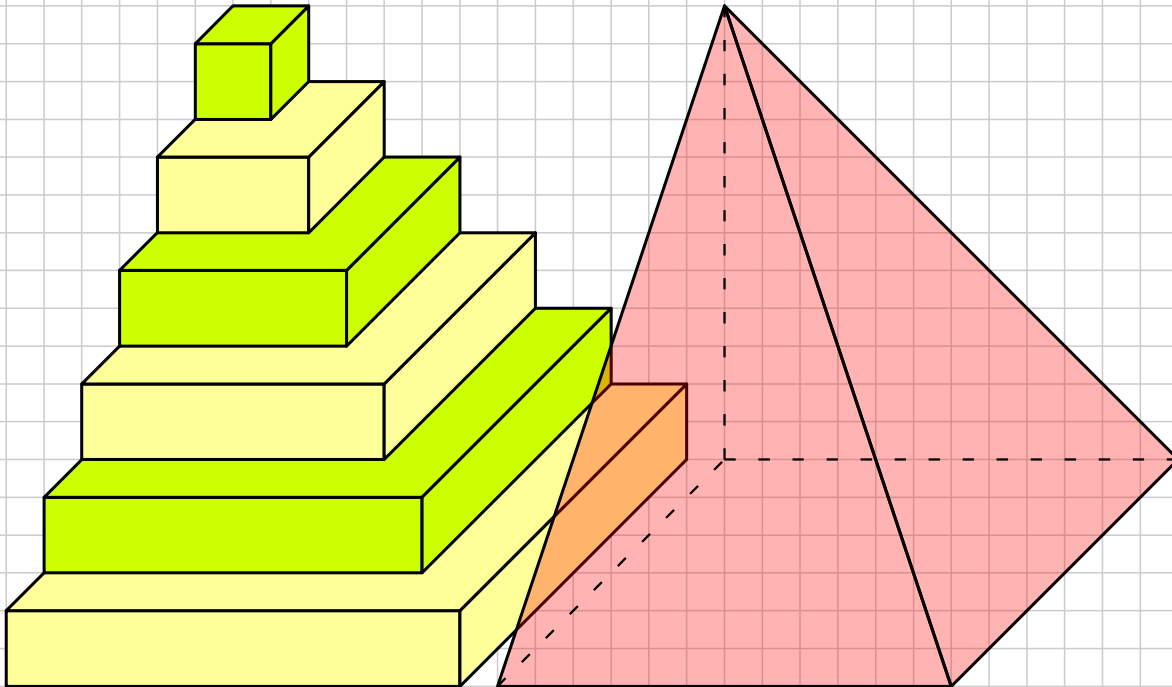
# Somma dei quadrati dei numeri interi



Somma dei quadrati dei numeri interi  
vista come piramide.



Somma dei quadrati dei numeri interi  
vista come piramide.



# Formula, e verifica in alcuni casi.

$$S_n = n^3/3 + n^2/2 + n/6 \quad \text{bella da vedere}$$

$$= n(2n^2 + 3n + 1)/6 \quad \text{comoda da calcolare}$$

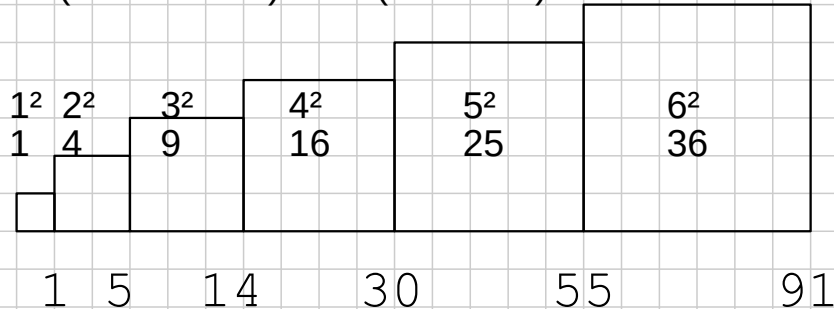
$$S_0 = 0 + 0 + 0$$

$$S_1 = 1/3 + 1/2 + 1/6 = 2/6 + 3/6 + 1/6 = 6/6 = 1$$

$$S_2 = 2*(2*4 + 3*2 + 1)/6 = 2*15/6 = 5$$

$$S_3 = 3*(2*9 + 3*3 + 1)/6 = 3*28/6 = 14$$

$$S_4 = 4*(2*16 + 3*4 + 1)/6 = 4(32 + 12 + 1)/6 = 30$$



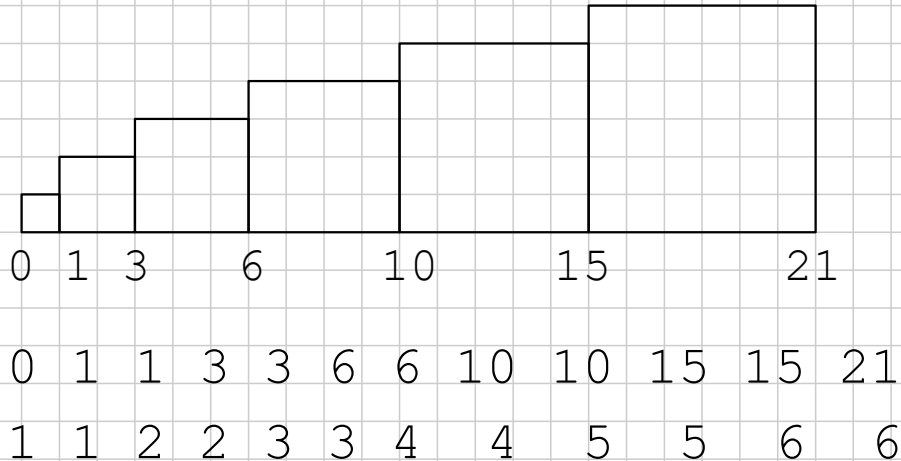
$$1^2 + 2^2 + 3^2 + \dots + n^2 = S_n$$

dimostrazione

dimostrazione

Provo con la formula del politrapezio.

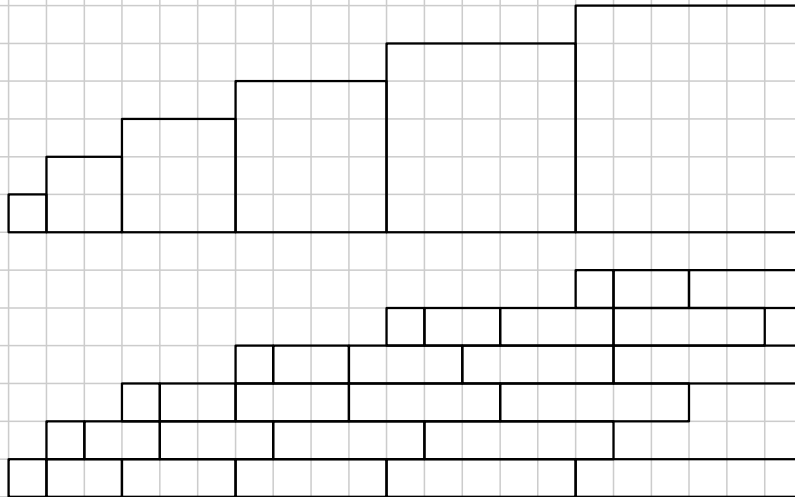
Vicolo cieco.



dimostrazione

Provo ad approssimare con un'altra figura.

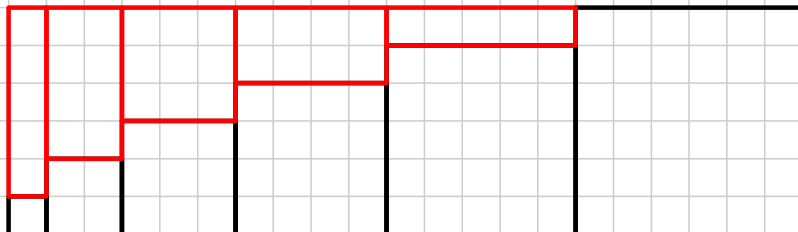
Vicolo cieco.

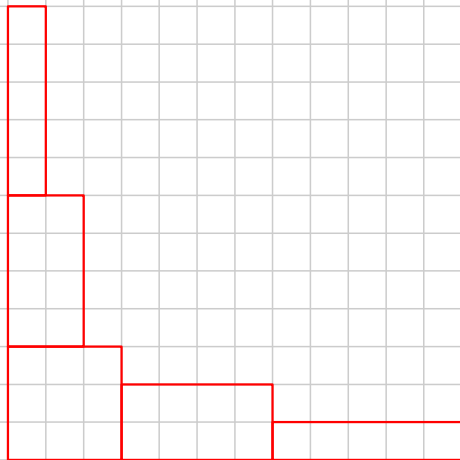




Studio la figura

- 1) la completo a un qualcosa di misurabile
- 2) forse la figura complementare e' piu' facilmente misurabile

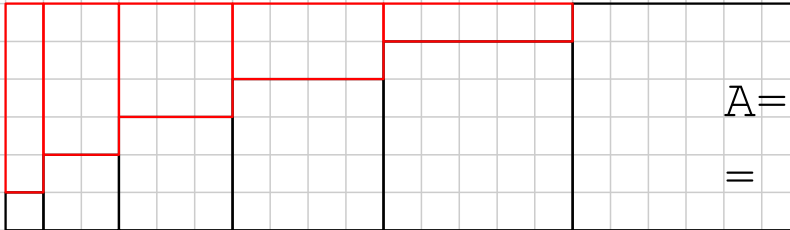
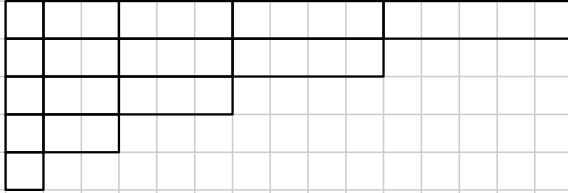




| n | Tn |    |  |  |  |
|---|----|----|--|--|--|
| 5 | 35 | 15 |  |  |  |
| 4 | 20 | 10 |  |  |  |
| 3 | 10 | 6  |  |  |  |
| 2 | 4  | 3  |  |  |  |
| 1 | 1  | 1  |  |  |  |

$$\begin{aligned}
 T_n &= (1^2 + 1 + 2^2 + 2 + 3^2 + 3 + \dots + n^2 + n) / 2 \\
 &= (1^2 + 2^2 + 3^2 + \dots + n^2) / 2 \\
 &\quad + (1 + 2 + 3 + \dots + n) / 2 \\
 &= S_n / 2 + (n^2 + n) / 4
 \end{aligned}$$

$$\begin{aligned}
 T(n-1) &= (S_n - n^2) / 2 + ((n-1)^2 + n - 1) / 4 \\
 &= S_n / 2 - n^2 / 2 + (n^2 + 1 - 2n + n - 1) / 4 \\
 &= S_n / 2 - n^2 / 4 - n / 4 \\
 &= S_n / 2 - (n^2 + n) / 4
 \end{aligned}$$



$$A = \text{base} * \text{altezza}$$

$$= ((n^2 + n) / 2) * n$$

$$A_n = S_n + T(n-1)$$

$$((n^2 + n) / 2) * n = S_n + S_{n-1} - n^2 / 4 - n / 4$$

$$(3/2) S_n = ((n^2 + n) / 2) * n + n^2 / 4 + n / 4$$

$$(3/2) S_n = (1/2) (n^3 + n^2 + n^2 / 2 + n / 2)$$

$$S_n = n^3 / 3 + n^2 / 2 + n / 6$$